NEWTON'S LAWS OF MOTION 6.0 Introduction

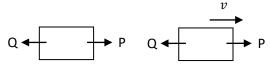
The object can be at rest, or in motion at constant velocity, or constant acceleration. See Physics for stating these Newton's laws.

Implications of Newton's 3 laws

1. Inertia: Reluctance to change the state. Resultant force F=0, a=0

At rest

Constant velocity



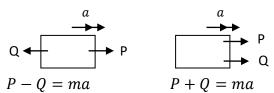
$$P - Q = 0 \Rightarrow P = Q$$

External forces P and Q balance.

Object remains at rest or continues with constant velocity v.

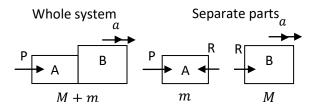
Constant velocity is shown with letter \boldsymbol{v} above one arrow.

2. Equation of motion: F = ma



Resultant force F causes acceleration a. Acceleration produced depends on the mass m eg 1 N on 1 Kg gives 1 ms⁻². Constant acceleration is shown with letter a above 2 arrows.

3. Action and Reaction: Equal and opposite.



P = (M + m)a, P - R = ma & R = MaSee Connected bodies (particles) later. Qn.1

(a) A block of mass 10 *Kg* rests on a table. What is the reaction of the table on the block?

- (b) A stone of mass 500 g is hanging on a string. What is the tension in the string?
- © Note:
- Internal forces R (action and reaction) are only shown on separate parts.
- 1st & 3rd laws may be considered to be special cases of 2nd law. Why? How?

Assumptions

Where necessary assume:

- 1. No air resistance to motion Motion through a vacuum.
- 2. No frictional force between surfaces Smooth surfaces (planes, pulleys & strings).
- 3. Light inextensible strings and light pulleys
- 4. Line of greatest slope of inclined planes

6.1 Resultant forces

Resultant force or accelerating force cause acceleration.

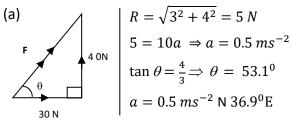
Resistances to motion are *external forces*. For example: frictional force, air resistance, gravitational force, resistance of obstacle to a bullet & braking force of a car.

Retarding forces cause deceleration (retardation). The object is gradually brought to rest.

Example 6.1

Find magnitude and direction of acceleration of an object of mass $10\ Kg$ acted on by forces $30\ N$ east and $40\ N$ north.

Soln:



Example 6.2

A car of mass 600 Kg initially at rest starts to move at constant acceleration. Find the resultant force if it acquires a velocity:

- (a) 20 ms⁻¹ in time 4 s
- (b) 12 ms⁻¹ through distance 24 m Assume no resistance to motion. Soln:
- (a) 1st equation of motion: v = u + at $20 = 0 + 4a \Rightarrow a = 4 \, ms^{-2}$ $F = ma = 600 \times 4 = 2400 N = 2.4 KN$
- (b) 3rd equation of motion: $v^2 = u^2 + 2as$ $12^2 = 0 + 2a \times 24 \implies a = 3 \text{ ms}^{-2}$ $F = ma = 600 \times 3 = 1800 N = 1.8 KN$

Example 6.3

A bullet of mass 20 g is fired into a wall with velocity 400 ms⁻¹. If the bullet penetrates the wall to a depth of 10 cm, find the resistance of the wall assuming it is uniform.

Soln:

3rd equation of motion:
$$v^2 = u^2 + 2as$$

 $0 = 400^2 + 2a \times 0.1 \Rightarrow a = -8 \times 10^5$
 $F = ma = 0.02 \times 8 \times 10^5 = 16 \ KN$

Vector form of Newton's 2nd law

F = ma

Where force F and acceleration a are vectors (bolded).

Example 6.4

- (a) An object of mass 2 Kg moves with acceleration 5i + 2j. Find force applied.
- (b) Two forces 9i 10j and -3i + 2j N act on a particle of mass 2 Kg. Find the magnitude of the acceleration.

Soln:

$$F = ma$$

(a)
$$\mathbf{F} = 2 {5 \choose 2} = 10 \mathbf{i} + 4 \mathbf{j}$$

(b)
$$\binom{9}{-10} + \binom{-3}{2} = 2a$$

$$\mathbf{a} = \binom{6}{-8}/2 = 3\mathbf{i} - 4\mathbf{j}$$

$$|\mathbf{a}| = \sqrt{3^2 + 4^2} = 8 \text{ ms}^{-2}$$

Exercise 6.1

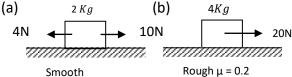
- 1. Find the magnitude and direction of acceleration if:
 - (a) a particle of mass 2 Kg is acted on by two forces $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $2\mathbf{i} +$ 5j + 2k.
 - (b) an object of mass 10 Kg is acted on by two forces 30 N east and 40 N N30°W.
- 2. A force 2i + 4j + ak acts on a particle and gives it acceleration i + 2j + 4k. Find:
 - (a) value of a and mass of the particle.
 - (b) magnitude of the force.

6.2 Force diagrams & Equations

Force diagrams show all the forces acting on an object.

See also Force diagrams in Chapter 2 earlier. Qn.2

Complete the force diagrams and find the accelerations of the objects.

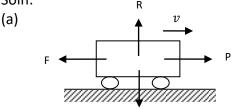


Example 6.5

Draw a force diagram. Hence apply Newton's laws of motion for a car moving at:

- (a) constant velocity along a smooth horizontal road under air resistance.
- (b) constant acceleration along a rough horizontal road.

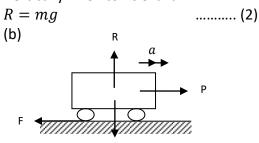
Soln:



mg

P = pull of engine (tractive force) F = air resistance (above the surface) Horizontally: Newton's 1st law

Vertically: Newton's 3rd law



P = pull of the engine (tractive force) F = frictional force (along the surfaces) Horizontally: Newton's 2nd law P - F = ma(1)

mg

Vertically: Newton's 3rd law

$$R = mg \qquad \qquad \dots (2)$$

Remember: $F = \mu R$

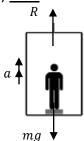
Example 6.6

A lift of mass M with a man of mass m is moving at constant acceleration upwards. Draw separate force diagrams and apply Newton's laws of motion for:

- (a) only the man.
- (b) the whole system (man and lift)
- (c) only the lift.

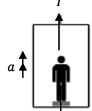
Soln:

(a) <u>Man</u>



R = reaction of lift on man Newton's 2nd law R - mg = ma $\Rightarrow R = m(a + g)$

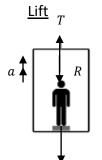
(a) Whole system



T = tension in the cord

Newton's 2nd law
$$T - (M + m)g = (M + m)a$$

$$\Rightarrow T = (M + m)(a + g)$$



T = tension in the cord R = reaction of man on lift Newton's 3rd law. R reversed. Newton's 2nd law T - R - Mg = Ma $\Rightarrow T = M(a + g) + R$

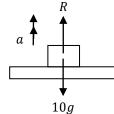
Example 6.7

A horizontal platform of mass 50 Kg has an object of mass 10 Kg fixed on top. It is pulled upwards by a cord and moves with constant acceleration 5 ms^{-2} . Find:

- (a) the reaction of the platform on the object.
- (b) the tension in the connecting cord. Soln:

This is similar to a man ascending in a lift.

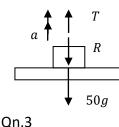
(a) <u>object</u>



R is the reaction of platform on object.

Newton's 2nd law $R - 10g = 10 \times 5$ $\Rightarrow R = 10(5 + 9.8) = 148 N$

(b) platform



T is the tension in the cord. R is the reaction of object on platform.

R reversed. Newton's 3rd law Newton's 2nd law $T - 148 - 50g = 50 \times 5$ $\Rightarrow T = 888 N$

What would be the reaction and tension if the acceleration was downwards?

Inclined planes

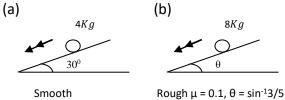
Forces are resolved parallel & perpendicular to the inclined plane. Why?

See Chapter 2 Statics of a particle.

Qn.4

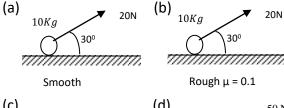
Complete the force diagrams of the objects. Hence find the accelerations of the objects.

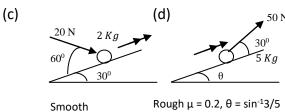
(M+m)gsmusingo@uict.ac.ug. Whatsapp: 0782506878 HoD ICT at UICT Plot 9-12 Port Bell Rd. Nakawa



Exercise6.2

1. Complete the force diagrams. Hence find the accelerations, sm





- 2. An object of mass 2 Kg lies on a rough inclined plane of angle arc sin 5/13 to the horizontal. A force 20 N is applied upwards parallel to the plane. If the object moves with acceleration 1.5 ms⁻², find the coefficient of friction.
- 3. A block of mass 8 *Kg* slides down a rough inclined plane of angle arc sin 1/6 to the horizontal. If the block moves with acceleration g/10, find the coefficient of friction.
- 4. A train moving on a level ground at 72 Km/h begins reaches a slope of 1 in 150 and the engine is switched off. If the total resistance to the motion is 1/160, find how far it goes before it comes to rest.

6.3 Connected bodies (particles)

Newton's 2nd and 3rd laws of motion are applied to a system of 2 or more bodies (particles).

Where necessary assume:

- 1. Smooth surfaces, pulleys & strings.
- 2. Light strings & pulleys.
- 3. Inextensible & taut strings.

Qn.5

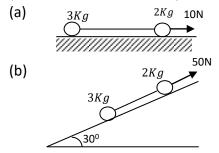
Draw force diagrams for the whole system and separate parts. Hence find the tensions.

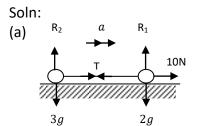


Example 6.8

Draw force diagrams, indicating directions of accelerations. Hence find the accelerations and the tensions.

(Assume smooth surfaces).





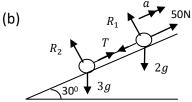
$$3 Kg: T = 3a$$
(1)

2 Kg: 10 - T = 2a....(2) 2 equations in 2 unknowns (T & a). Add to eliminate T:

$$5a = 10 \implies a = 2 \, ms^{-2}$$

Eqn 1:
$$T = 3 \times 2 = N$$

(b) $R_1 = 5$



Acceleration: Newton's 2nd law.

3 Kg:
$$T - 3gsin30^0 = 3a$$
(1)

2 Kg:
$$50 - T - 2gsin30^0 = 2a$$
(2)

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2 equations in 2 unknowns (T & a).

Add to eliminate T:

$$5a = 50 - 5gsin30^{\circ} \Rightarrow a = 5.1 \, ms^{-2}$$

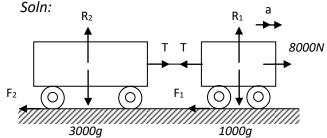
Eqn 1:
$$T = 3(5.1 + gsin30^{\circ}) = 30 N$$

Qn.6

Repeat the above if the surfaces are rough and $\mu = 0.1$.

Example 6.9

An engine of mass $1\times 10^3~Kg$ pulls a coach of mass $3\times 10^3~Kg$ exerting a pull $8\times 10^3~N$. The coefficient of friction is 1/5. Find their acceleration and the tension in the connecting cord.



Engine:

$$8000 - \frac{1}{5} \times 1000g - T = 1000a$$
 (1)

Coach:

$$T - \frac{1}{5} \times 3000g = 3000a$$
 (2)

Combined:

$$8000 - \frac{1}{5} \times 4000g - T = 4000a$$
 (3)

Note:

2 unknowns (a & T), but 3 equations.

therefore choose only 2 equations.

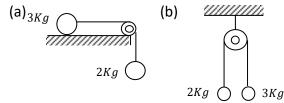
(1) & (2): Add to eliminate $T \Rightarrow a = 0.04 \text{ m/s}^2$

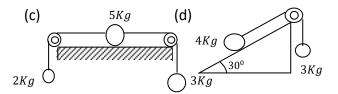
(1) or (2): Substitute for a. \Rightarrow T = 6 x 10³ N

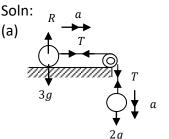
Example 6.10

Draw force diagrams, indicating possible directions of accelerations. Hence find the accelerations, tensions and reactions on the pulleys.

Where necessary make assumptions.







$$3 Kg: T = 3a$$
(1

$$2 Kg: 2g - T = 2a$$
(2)

2 equations in 2 unknowns (T & a).

Add to eliminate T:

$$5a = 2g \implies a = \frac{2g}{5} = 3.92 \text{ ms}^{-2}$$

Eqn 1:
$$T = 6g/5 \simeq 11.76 N$$

Check: Use Eqn 2.

Reaction R on pulley.

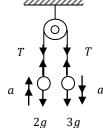
Resultant of tensions T (at 90°)

Vector method, $R = \sqrt{T^2 + T^2} = \sqrt{2} T$

Alt:

Resolution, $R = 2T\cos 45^{\circ} = \sqrt{2} T$ $R = \sqrt{2} \times 6g/5 \simeq 16.63 N$, 45° to horiz.

 $R = \sqrt{2} \times 6g/5 \simeq 16.63 N, 45^{\circ} \text{ to}$ (b) ////////



$$3 Kg: 3g - T = 3a$$
(1)

$$2 Kg: T - 2g = 2a$$
(2)

2 equations in 2 unknowns (T & a).

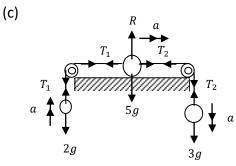
Add to eliminate T:

$$5a = g \implies a = \frac{g}{5} \simeq 1.96 \text{ ms}^{-2}$$

Eqn 2:
$$T = 2g(1 + \frac{1}{5}) = \frac{12g}{5} \approx 23.52 \, N$$

Reaction R on pulley: Resultant of tensions T (downwards).

$$R = \frac{24g}{5} \simeq 47.04 N$$
, downwards.



$$2 Kg: T_1 - 2g = 2a$$
(1

$$5 Kg: T_2 - T_1 = 5a$$
(2)

$$3 Kg: 3g - T_2 = 3a$$
(3)

3 equations in 3 unknowns (T_1 , $T_2 \& a$).

Add to eliminate
$$T_1 \& T_2$$
: $10a = g$

$$a = g/10 \simeq 0.98 \, ms^{-2}$$

Eqn 1:
$$T_1 = 2g\left(1 + \frac{1}{10}\right) = \frac{22g}{10} \approx 21.56 \, N$$

Eqn 3:
$$T_2 = 3g \left(1 - \frac{1}{10}\right) = \frac{27g}{10} \approx 26.46 \, N$$

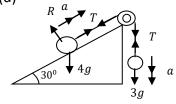
Reactions ($R_1 \& R_2$) on pulleys: Similar to part (a) above.

$$R_1 = 2T_1 \cos 45^0 = \sqrt{2} \, T_1$$

$$R_1 = \sqrt{2} \cdot 22g/10 = 30.49 N$$
, 45° to horiz.

$$R_2 = 2T_2 cos 45^0 = \sqrt{2} T_2$$

$$R_2 = \sqrt{2} \cdot 27g/10 = 37.42 \, N$$
, 45° to horiz.



3 Kg:
$$3g - T = 3a$$
(1

4 Kg:
$$T - 4gsin30^0 = 4a$$
(2)

2 equations in 2 unknowns (T & a).

Add to eliminate T:
$$7a = g$$

 $a = \frac{g}{7} = 1.40 \text{ ms}^{-2}$

Eqn 1:
$$T = 6g/5 \simeq 11.76 N$$

Check: Use Eqn 2.

Reaction R on pulley:

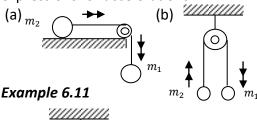
Resultant of tensions T (at 60°)

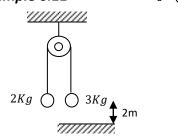
Resolution:
$$R = 2T\cos 30^{\circ} = \sqrt{3} T$$

$$R = \sqrt{2} \cdot 6g/5 \simeq 16.63 \, N$$
, 30° to vert.

Qn.7

Complete the force diagrams and apply Newton's laws of motion. Hence find expressions for accelerations.





Masses 2 Kg and 3 Kg are connected by a string passing over a pulley. The masses are released from rest at a height 2 m from the ground. Find:

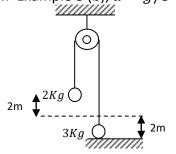
- (a) acceleration and greatest height reached by 2 Kg mass.
- (b) speed of 3 Kg mass on reaching the ground and time taken.

Where necessary make assumptions.

Soln:

(a)

Assume 2 Kg mass does not reach the pulley. From Example 8 (b), $a=g/5\simeq 1.96~ms^{-2}$



1st (string taut):

2 Kg accelerates & covers 2 m upwards.

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 1.96 \times 2 \implies v = 2.8 \, \text{ms}^{-1}$$

2nd (string slack):

2 Kg decelerates & comes to rest.

$$v^2 = u^2 + 2as$$

$$0 = 2.8^2 + 2 \times 9.8s \implies s = 0.4 m$$

Total height =
$$2 + 2 + 0.4 = 4.4 m$$
 (b)

Assume 3 Kg mass does not rebound from the ground.

String taut:

3 Kg accelerates & covers 2 m to ground. Final velocity $v = 2.8 ms^{-1}$ (same as 2 Kg)

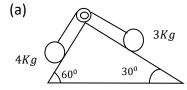
$$v = u + at$$

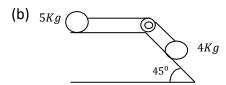
$$2.8 = 0 + 1.96t \Rightarrow t = 1.43 s$$

Exercise 6.3

Where necessary make assumptions.

1. Draw force diagrams. Hence find the accelerations of masses, tensions in strings and reactions on pulleys.





2. A mass 10 *Kg* lies on a smooth horizontal surface. A string from it passes over a fixed pulley 1.5 m away at the edge to a mass 8 Kg hanging below.

The masses are released from rest. Find:

- (a) acceleration of the masses, tension in the string and reaction on the pulley.
- (b) speed of the masses when 10 Kg reaches the pulley.

- 3. A mass 5 Kg lies on a smooth inclined plane of angle 30° to the horizontal. A string from it passes over a fixed pulley 3 m away at the top to a mass 3 Kg at 2 m above the ground. The masses are released from rest. Find:
 - (a) acceleration of the masses, tension in the string and reaction on the pulley.
 - (b) speed of the masses when 3 Kg reaches the ground.
 - (c) distance covered afterwards by 5 *Kg* when it first comes to rest.

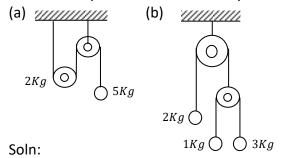
Related accelerations

Connected bodies may move with different accelerations, but related.

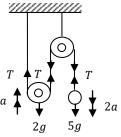
Example 6.12

Draw force diagrams. Hence find the accelerations of masses, tensions in strings and reactions on pulleys.

Make assumptions where necessary.



(a) If movable pulley goes up distance x, 5 Kg mass goes down distance 2x. Why? Let a= acceleration of movable pulley Then, 2a = acceleration of 5 Kg mass



 $5 Kg \text{ mass: } 5g - T = 5 \times 2a$ (1)

2 Kg pulley: 2T - 2g = 2a(2)

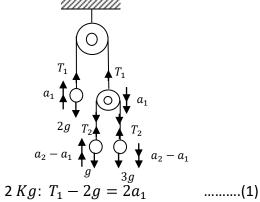
2 equations in 2 unknowns (T & α).

Combined: Add 2(1) & (2): $a = 4g/11 \simeq 3.56 \ ms^{-2}$ Eqn 2: $T = 3.56 + 9.8 \simeq 13.36 \ N$

(b) Assume lower pulley is light ($m \simeq 0$).

Let, a_1 = acceleration of 2 Kg & lower pulley a_2 = acc of 3 Kg & 1 Kg rel. to lower pulley Since $\frac{3}{1} > \frac{4}{2}$ (ratios of masses) $a_2 > a_1$.

 a_2-a_1 = acceleration of 3 Kg & 1 Kg



Movable pulley: $2T_2 - T_1 = 0$ (2)

$$3 Kg: 3g - T_2 = 3(a_2 - a_1)$$
(3)

$$1 Kg: T_2 - g = 1(a_2 - a_1)$$
(4)

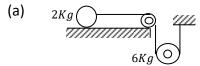
Note:

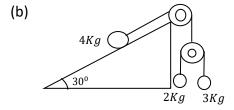
4 equations in 4 unknowns (T_1 , T_2 , a_1 & a_2). Qn.8 Complete to find the 4 unknowns.

Exercise 6.4

Draw force diagrams. Hence find the accelerations of masses, tensions in strings and reactions on pulleys.

Make assumptions where necessary.





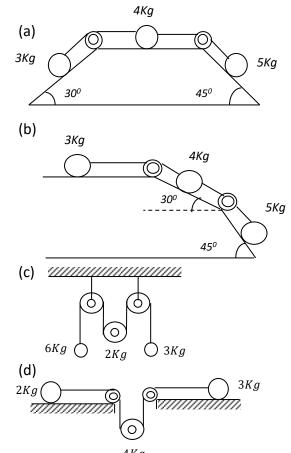
Revision Exercises

Exercise 6A

Where necessary make assumptions.

- 1. Two forces 10i + 9j and ai + bj act on a particle of mass 500 g. If acceleration is 24i + 3j. Find values of a and b.
- 2. Forces i 2j and 3i + 4j act on a particle of mass 2 Kg. Initially the particle is at a point with position vector 2i j and moving with velocity 4i + 3j. Find the time when the position vector and acceleration are in the same direction.
- 3. A car of mass 600 Kg moving with constant acceleration changes its velocity from 72 Kmh⁻¹ to 144 Kmh⁻¹ in 10 s. Find:
 - (a) the accelerating force if there is no resistance to motion.
 - (b) the pull of the engine if the resistance to motion is 100 N.
- 4. A bullet of mass 30 g is fired into a fixed block of wood with velocity 294 ms⁻¹ and bought to rest in 1/150 s. Find resistance exerted by the wood, assuming it is uniform.
- 5. A man of mass $100 \, Kg$ stands in a lift which moves in three stages. First, acceleration at $1 \, m/s^2$ to constant velocity. Second, constant velocity for a given time. Third, retardation at $1.2 \, m/s^2$ to rest. Find the reactions on the man at each stage.
- 6. A train of mass 225×10^3 Kg moving on a level ground at 72 Km/h begins to go up a slope of 1 in 75. The tractive force of the engine is 24.5 KN and total resistance to motion is 14.7 KN. Find how far the train climbs before it comes to a stand still.
- 7. A break down car of mass $1 \times 10^3~Kg$ tows a car of mass 600~Kg up a road which rises 1~in~20. Frictional forces are 0.1 N per Kg. Find their acceleration and the tension in the tow bar.

8. Draw force diagrams. Hence find the accelerations of masses, tensions in strings and reactions on pulleys.



- 9. A mass 8 Kg lies on a rough horizontal surface. The coefficient of friction is 1/2. A string from it passes over a fixed pulley 2 m away at the edge to a mass 10 Kg hanging 1.5 m above the ground. The masses are released from rest. Find:
 - (a) acceleration of masses, tension in the string and reaction on the pulley.
 - (b) speed of the masses when 10Kg reaches the ground.
 - (c) speed of the 8 Kg on reaching the pulley.
- 10. A mass 3 Kg lies on a rough inclined plane of angle 30^0 to the horizontal. The coefficient of friction is 1/3.

A string from it passes over a fixed pulley 2.5 m away at the top to a mass 5 Kg at 2 m above the ground. The masses are released from rest. Find:

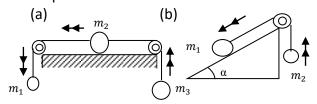
- (a) acceleration of the masses, tension in the string and reaction on the pulley.
- (b) speed of masses when 3 Kg reaches the ground.
- (c) speed of 5 Kg mass on reaching the pulley.

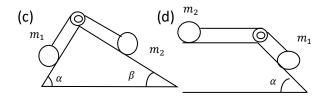
Exercise 6B

Where necessary make assumptions.

- 1. A force $2t\mathbf{i} + t\mathbf{j} + 3t\mathbf{k}$ acts on a particle of mass 2 Kg at time t. If the particle is initially at rest, find acceleration at time t and velocity after 3 s.
- A lift of mass M with an object of mass m hanging from the roof is moving downwards. Draw separate force diagrams for the object and the lift, and apply Newton's laws of motion for:
 - (a) constant velocity.
 - (b) constant acceleration.
- 3. A car of mass m is moving upwards along an inclined plane of angle θ to the horizontal. Draw a force diagram and apply Newton's laws of motion for constant acceleration if the surface is:

 (a) smooth (b) rough
- 4. Complete the force diagrams and apply Newton's laws of motion. Hence find expressions for the accelerations.





CHAPTER 6

- 5. A ball rolls from rest down a rough plane inclined at 30° to a smooth horizontal plane. The coefficient of friction is 0.25. On reaching the horizontal plane, find velocity of the ball and time taken to move 2 m. (UNEB 1989)
- 6. A train of mass 300 *tonnes* is pulled from rest down an incline of by an engine of tractive force 120 *KN*. The air resistance to motion is 90 *KN*.
 - (a) If the train attains a speed of $63 \ Km/h$ in 25 s, find the slope of the incline.
 - (b) If the driver then applies brakes on seeing an object in front on the rails and the train stops at distance $300\ m$ a head, find the force exerted by the brakes.
 - (c) If the brakes fail and the train drags the object along at constant speed $25 \, Km/h$, find the resistance to the motion.

(UNEB 1991)